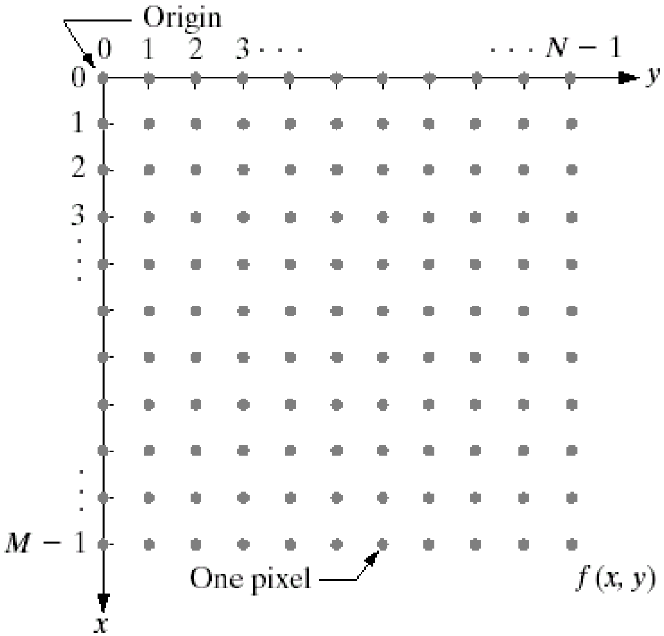
Basic Relationships between Pixels and Distance Measures

M. N. Tibdewal, Ph.D.(IIT, Kharagpur)
Department of Electronics & Telecom. Engg.
Shri Sant Gajanan Maharaj College of Engg.,Shegaon

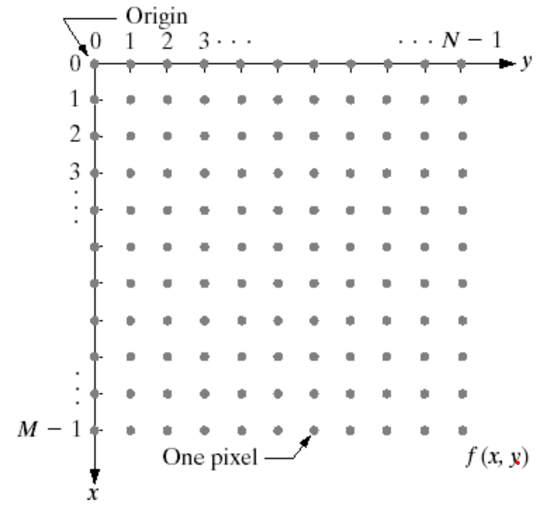
Some Basic Relationships Between Pixels

- Definitions:
  - f(x,y): digital image
  - Pixels: q, p
  - Subset of pixels of f(x,y): S



Neighbors of a Pixel

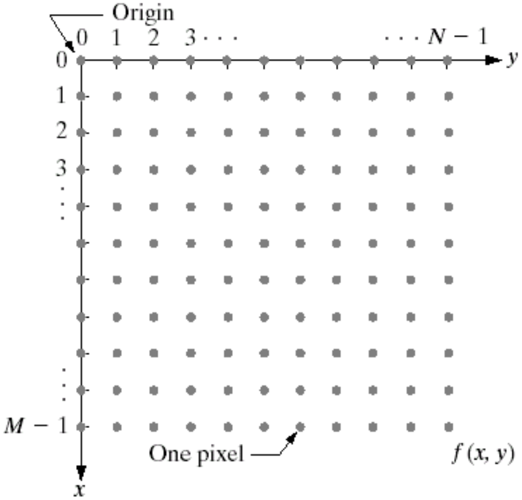
- A pixel p at (x,y) has 2 horizontal and 2 vertical neighbors:
  - (x+1,y), (x-1,y), (x,y+1),(x,y-1)



  - This set of pixels is called the 4-neighbors of p: $N_4(p)$
  - Each pixel is a unit distance from (x,y)
  - Some of the neighbor locations of ‘p’ lie outside the image if (x,y) is on border of the image.

Neighbors of a Pixel

- The 4 diagonal neighbors of p are: ($N_D(p)$)
  - (x-1,y-1), (x-1,y+1),
  - (x+1,y-1), (x+1,y+1)



- $N_4(p) + N_D(p) \rightarrow N_8(p)$: the 8-neighbors of 'p'
- Some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x,y) is on border.

Connectivity

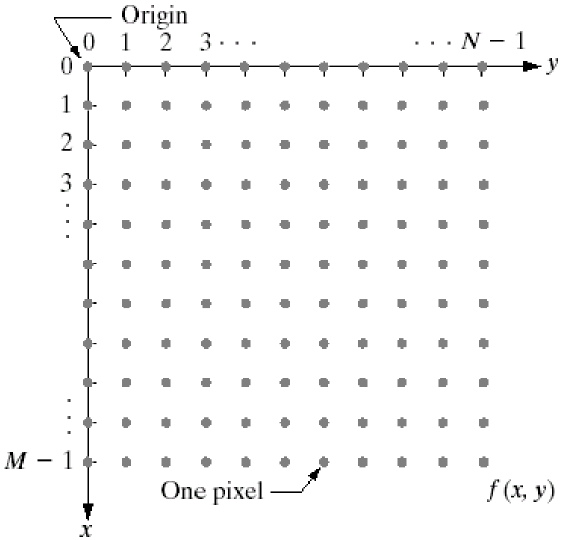
- Connectivity between pixels is important:
  - Because it is used in establishing boundaries of objects and components of regions in an image

Connectivity…

- Two pixels are connected if:
  - They are neighbors (i.e. adjacent in some sense -- e.g. $N_4(p)$, $N_8(p)$, …)
  - Their gray levels satisfy a specified criterion of similarity (e.g. equality, …)
- V is the set of gray-level values used to define adjacency (e.g. V={1} for adjacency of pixels of value 1)

Adjacency

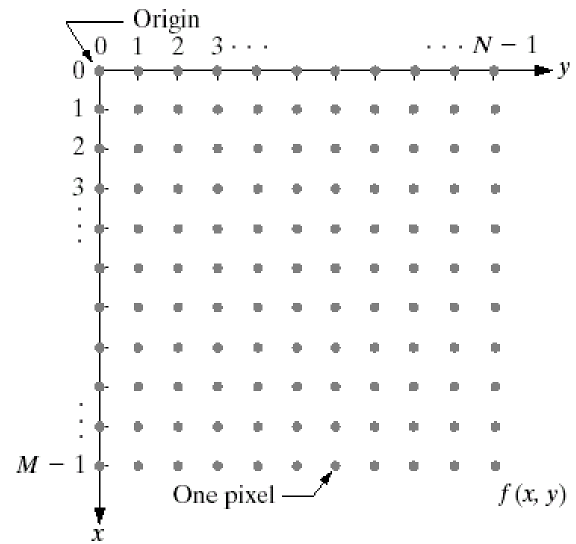
- We consider three types of adjacency:
  - 4-adjacency: two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$



  - 8-adjacency : p & q are 8- adjacent if q is in the set $N_8(p)$

Adjacency

- The third type of adjacency:
  - Mix(m) adjacency: p & q with values from V are m-adjacent if



- q is in $N_4(p)$ or q is in $N_D(p)$ and
- the set $N_4(p) \cap N_4(q)$ has no pixels with values from V

For example:

(i) q is in $N_4(p)$

0	1	0
1	1	1
0	1	0

(ii) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels with values from V

1	0	1
0	1	0
1	0	1

Adjacency

- Mixed adjacency is a modification of 8-adjacency and is used to eliminate the multiple path connections that often arise when 8-adjacency is used.

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

Sample arrangement of pixels

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent (Ambiguous?)

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

m-adjacent

Adjacency

- Two images subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.

Two Regions that are adjacent if 8 adjacency is used:

$$\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right\}\text{R1}$$

$$\left.\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\}\text{R2}$$

Path

- A path (curve) from pixel p with coordinates (x,y) to pixel q with coordinates (s,t) is a sequence of distinct pixels:
  - (x_0,y_0), (x_1,y_1), ..., (x_n,y_n)
  - where $(x_0,y_0)=(x,y)$, $(x_n,y_n)=(s,t)$, and (x_i, y_i) is adjacent to (x_{i-1},y_{i-1}), for $1\leq i \leq n$; **n is the length of the path**.
- If $(x_0, y_0) = (x_n, y_n)$: A closed path

The inner boundary of the **‘1’valued region** does not form a closed path, but its outer boundary **(‘0’ valued)** form a close path, For e.g.:

0	0	0
0	1	0
0	1	0
0	1	0
0	1	0
0	0	0

Paths

- 4-path, 8-path, and m-path can be defined depending on the type of adjacency specified.

- If $p, q \in S$, then q is connected to p in S if there is a path from p to q consisting entirely of pixels in S.

Connectivity

- For any pixel p in S, the set of pixels in S that are connected to p is a connected component of S.

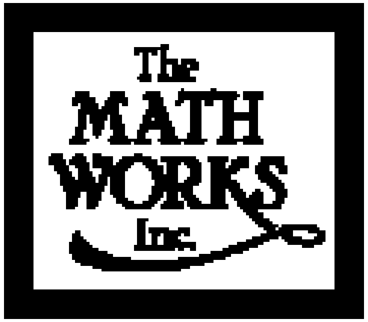
- If S has only one connected component then S is called a connected set.

Boundary

- R a subset of pixels in an image:

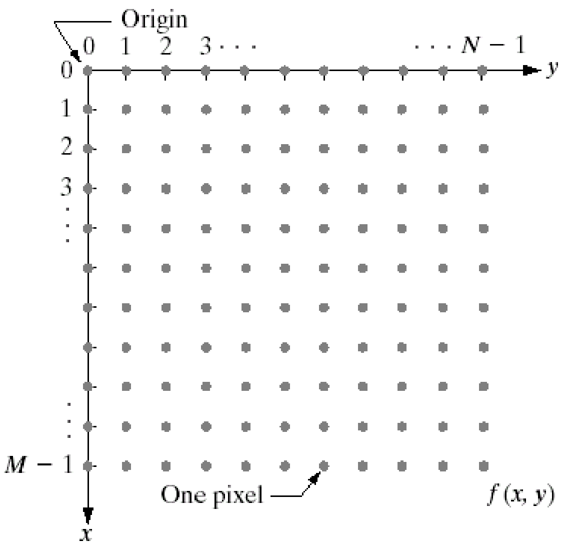
  - R is a region if R is a connected set.

- Its boundary (border/contour) of a region R is the set of pixels in region R that have at least one or more neighbors not in R.
- Edge can be the region boundary (in binary images)



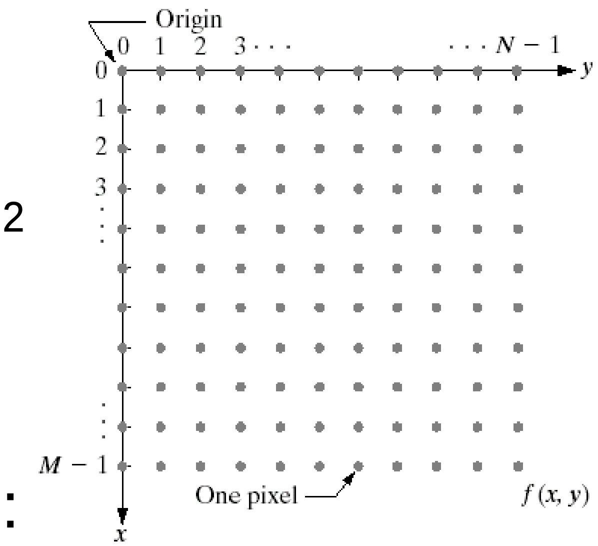
Distance Measures

- For pixels p,q,z with coordinates (x,y), (s,t), (u,v), D is a distance function or metric if:
  - $D(p,q) \geq 0$;($D(p,q)=0$ if $p=q$)
  - $D(p,q) = D(q,p)$; and
  - $D(p,z) \leq D(p,q) + D(q,z)$



Distance Measures…

- Euclidean distance:
  - $D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$



  - For this distance measure: pixels (points) having a distance less than or equal to r from (x,y) are contained in a disk of radius r centered at (x,y).

Distance Measures…

- D_4 distance (city-block distance):
  - $D_4(p,q) = |x-s| + |y-t|$
  - forms a diamond centered at (x,y)
  - e.g. pixels with $D_4 \leq 2$ from p,

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

$D_4 = 1$ are the 4-neighbors of p

Distance Measures…

- D_8 distance (chessboard distance):
  - $D_8(p,q) = \max(|x-s|,|y-t|)$
  - Forms a square centered at p
  - e.g. pixels with $D_8 \leq 2$ from p,

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

$D_8 = 1$ are the 8-neighbors of p

Distance Measures…

D_4 and D_8 distances between p and q are independent of any paths that exist between the points because these distances involve only the coordinates of the points...

(regardless of whether a connected path exists between them).

Distance Measures…

- However, for m-connectivity the value of the distance (length of path) between two pixels depends on the values of the pixels along the path and those of their neighbors.

Distance Measures…

- e.g. assume p, p_2, $p_4 = 1$

  p_1, p_3 = can have either 0 or 1

	p_3	p_4
p_1	p_2	
p		

If only connectivity of pixels valued 1 is allowed, and p_1 and p_3 are 0, the m-distance between p and p_4 is 2.

If either p_1 or p_3 is 1, the distance is 3.

If both p_1 and p_3 are 1, the distance is 4 ($pp_1p_2p_3p_4$)

Hum Badlenge Yug Badlega

Hum Sudhrenge Yug Sudhrega

The greatest gift you give
someone is your time.
Because when you give your
time you are giving a portion
of your life that you will
never get back.

All World Gayatri Priwar
Shantikunj Haridwar

Questions Please ?

Thanks!