★ Slant Transform

Slant transform was introduced by Enomoto and Shibata as an 'orthogonal' transform containing discrete sawtooth waveforms or 'slant basis vectors'.

- Specifically designed for image coding.
- A Slant basis vector that is monotonically decreasing in constant steps from maximum to minimum has the sequency property and has a fast computational algorithm.
- A slant basis vector efficiently represents linear brightness variations along an image line.
- The slant transformation has been utilize in several transform image-coding systems for monochrome and color images.
- Let S_N denote $N \times N$ slant matrix with $N = 2^n$, then it is defined by recursion.

$2^1 = N$; $n = 1$.

$$S_2 = S_{2^1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

where $I_{N'} \rightarrow N' \times N'$ Identity Matrix

$$\underset{N=2^n}{S} = \frac{1}{\sqrt{2}}\left[\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ a_n & b_n & & -a_n & b_n & \\ \hline \multicolumn{2}{c|}{0} & I_{\left(\frac{N}{2}\right)-2} & \multicolumn{2}{c|}{0} & I_{\left(\frac{N}{2}\right)-2} \\ \hline 0 & 1 & 0 & 0 & -1 & 0 \\ -b_n & a_n & & b_n & a_n & \\ \hline \multicolumn{2}{c|}{0} & I_{\left(\frac{N}{2}\right)-2} & \multicolumn{2}{c|}{0} & -I_{\left(\frac{N}{2}\right)-2} \end{array}\right]\left[\begin{array}{c|c} S_{n-1} & 0 \\ \hline 0 & S_{n-1} \end{array}\right]$$

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Slant matrix.

The parameter a_n, b_n are defined by the recursions :-

$$a_1 = 1 \qquad b_n = \left(1 + 4a_{n-1}^2\right)^{-\frac{1}{2}}$$

$$a_n = 2 b_n a_{n-1}$$

$$a_{n+1} = \left(\frac{3N^2}{4N^2-1}\right)^{\frac{1}{2}} \qquad b_{n+1} = \left(\frac{N^2-1}{4N^2-1}\right)^{\frac{1}{2}}$$

where $\underline{N = 2^n}$

Using above formulae the $\underline{4 \times 4}$ slant transform matrix is obtained as.

$$S_{N=2^2} = S_4 = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{array}{c} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} \\ 1 & -1 & -1 & 1 \\ 1/\sqrt{5} & -3/\sqrt{5} & \frac{3}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{array}\right] \end{array} \begin{array}{c} \underline{\text{sequency}} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

$n = 2, \; \underline{N = 4}$

$$S_N = S_{2^n = N} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ a_2 & b_2 & -a_2 & b_2 \\ 0 & 1 & 0 & -1 \\ -b_2 & a_2 & b_2 & a_2 \end{bmatrix} \begin{bmatrix} S_{2-1} & 0 \\ 0 & S_{2-1} \end{bmatrix}$$

$\uparrow$ 4 $\quad 2^2 = 4$

$$a_1 = 1 \qquad b_2 = \left(1 + 4a_1^2\right)^{-\frac{1}{2}} = (1+4)^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$$

$$a_2 = 2 b_2 a_1 = 2 \cdot \frac{1}{\sqrt{5}} \cdot 1 = 2/\sqrt{5} = a_2$$

$$S_{N=4} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2/\sqrt{5} & \frac{1}{\sqrt{5}} & -2/\sqrt{5} & \frac{1}{\sqrt{5}} \\ 0 & +1 & 0 & -1 \\ -1/\sqrt{5} & +2/\sqrt{5} & 1/\sqrt{5} & \frac{+2}{\sqrt{5}} \end{bmatrix}_{4\times4} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}_{4\times4}$$

$$S_{N=4} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & 1/\sqrt{5} & -\frac{1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} \\ 1 & -1 & -1 & 1 \\ 1/\sqrt{5} & -\frac{3}{\sqrt{5}} & \frac{3}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix} \underline{4\times4}$$

$\Leftarrow$ {4x4 matrix will be generated}

$$S_{\underset{8=N}{N}} = S_{\underset{2^n=N}{}} = S_8 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ a_3 & b_3 & 0 & 0 & -a_3 & b_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ -b_3 & a_3 & 0 & 0 & b_3 & a_3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}_{8\times 8} = \begin{bmatrix} S_2 & 0 \\ 0 & S_2 \end{bmatrix}_{8\times 8}$$

$(n-1)$ $n-1$

$\boxed{n=3}$; $2^3=8$

$\boxed{N=8}$

$I_{\left(\frac{8}{2}-2\right)} = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{I_1 = 1}$ $\boxed{I_0 = 0}$

$S_{N=8} = S_{n=3} = [8]\times[8]$ matrix.

*** Properties of 'slant' Transform:-**

1. The slant transform is real and orthogonal therefore, $S = S^*$; $S^{-1} = S^T$
2. The slant Trans is a fast transform, which can be implemented in $O(N\log_2 N)$ operations ~~as~~ an $N \times 1$ vector.
3. It has very good to excellent, energy compaction for images.
4. The basis vectors of the 'Slant' Trans matrix 'S' are not sequency ordered. for $n \geq 3$, If S_{n-1} is sequency ordered, the ith row sequency of S_n is given as follows:-

$i=0$, sequency $=0$

$i=1$; sequency $=1$

$2 \leq i \leq \left(\frac{N}{2}\right)-1$; sequency $= \begin{cases} 2i \ ; & i\text{ even} \\ 2i+1 \ ; & i\text{ odd} \end{cases}$

$i = \left(\frac{N}{2}\right)$; sequency 2.

$i = \left(\frac{N}{2}+1\right)$; sequency $=3$

$\left(\frac{N}{2}+2\right) \leq i \leq (N-1)$; Sequency $= \begin{cases} 2\left(i-\frac{N}{2}\right)+1 \ ; & i\text{-even} \\ 2\left(i-\frac{N}{2}\right) \ ; & i\text{-odd} \end{cases}$