★ Hadamard Transforms :-

The elements of the basis vectors of the Hadamard transform take only the binary ± 1 and are there well suited for Digital signal processing.

The Hadamard transform matrices, 'H_n' are $N \times N$ matrices,

where $N = 2^n$; $n = 1, 2, 3$,

these can be easily generated by the core matrix,

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{---} \quad (1)$$

and the Kronecker Product Recursion

$$H_n = \underbrace{H_{n-1} \otimes H_1}_{} = \underbrace{H_1 \otimes H_{n-1}}_{}$$

$$H_n = \frac{1}{\sqrt{2}} \begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix}$$

for eg

$H_3 = H_2 \otimes H_1$

$\underbrace{H_2 = H_1 \otimes H_1}$

$$H_2 = H_1 \otimes H_1 = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix}$$

$$H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix}$$

→ It is convinent for storage/transmission purpose.

→ The elements of mutually orthogonal basis vectors of a Hadamard transform are either $+1$ or -1.

→ It results is very low computational complexity in the calculation of the transform coefficients.

→ 2D-Hadamard

$$H(u,v) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \underbrace{f(x,y)}_{\text{(Image)}} \underbrace{\left[\prod_{i=0}^{n-1} (-1)^{b_i(x) b_i(u) + b_i(y) b_i(v)} \right]}_{\text{Kernel/Basic func of Hadamard Transform}}$$

N - order of Matrix

(Forward 2D-Hadamard Transform) or

→ 2D inverse Hadamard Transform

$$f(x,y) = \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} H(u,v) \underbrace{\left[\prod_{i=0}^{n-1} (-1)^{b_i(x) b_i(u) + b_i(y) b_i(v)} \right]}_{\text{Kernel/Basis function}}$$

for $H_3 = H_2 \otimes H_1 = H_1 \otimes H_1 \otimes H_1$.

$$H_3 = \frac{1}{\sqrt{8}} \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & +1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{array} \right]_{8\times 8} \begin{array}{c} 0 \\ 7 \\ 3 \\ 4 \\ 1 \\ 6 \\ 2 \\ 5 \end{array}$$

sequency (zero-crossing)

The number of zero crossings (of a Walsh function) or the number of transitions in a basis vector of the Hadamard transform is called its 'sequency'.

* The existing 'sequency' order of these vectors is called Hadamard order.

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· Properties of Hadamard (or Walsh) Transform.

→ The basis vector of the Hadamard transform can also be generated by sampling a class of function called 'the Walsh' function.

→ The order of basis functions of Hadamard transform does not allow the fast computation of it by using a straight forward modification of FFT.

→ Most of the properties of 'Walsh' transform are valid with 'Hadamard' transform.

→ Hadamard Transform is basically the same as 'Walsh' Transform except the rows of the transform matrix are reordered. Rest all the properties are same.