✱ Discrete Cosine Transform (DCT):—

- DFT is very popular due to its good computational efficiency. DCT also gives us frequency domain from spatial domain, but different way.
- In some IP applications computational efficiency is not the issue, the complexity is the issue.
- DFT has complex calculations. DCT do not have complex nos, only real nos.
- DFT has poor energy compaction, etc.
- DCT has been developed by Ahmed Natarajan & Rao 1974.
- The DCT is the member of the family of real valued discrete sinusoidal unitary Transform.
- unitary $\Rightarrow A A^T = I$.
- The discrete cosine transform consists of a set of basis vectors that are cosine functions.
- DCT is a technique for converting a signal into elementary frequency components and is widely used in image compression.
- DCT is used in JPEG for compression. (It is very impt).
- The ability to pack energy of the spatial sequence into as few frequency coefficients as possible is called as Energy compaction.
- If compaction is high, we only have to transmit a few coefficients, instead of the whole of pixels.
- To perform the JPEG coding, an image (in color/gray scales) is first subdivided into blocks of 8x8 pixels.
- The discrete Cosine Transⁿ (DCT) is then performed on each block.
- This generates 64 coefficients which are then quantized to reduce their magnitude.

- The coefficients are then recorded into a one dimentional array in a zigzag manner before further entropy encoding.
- The compression is achieved in two stages: the first is during quantization and the second during entropy coding process.
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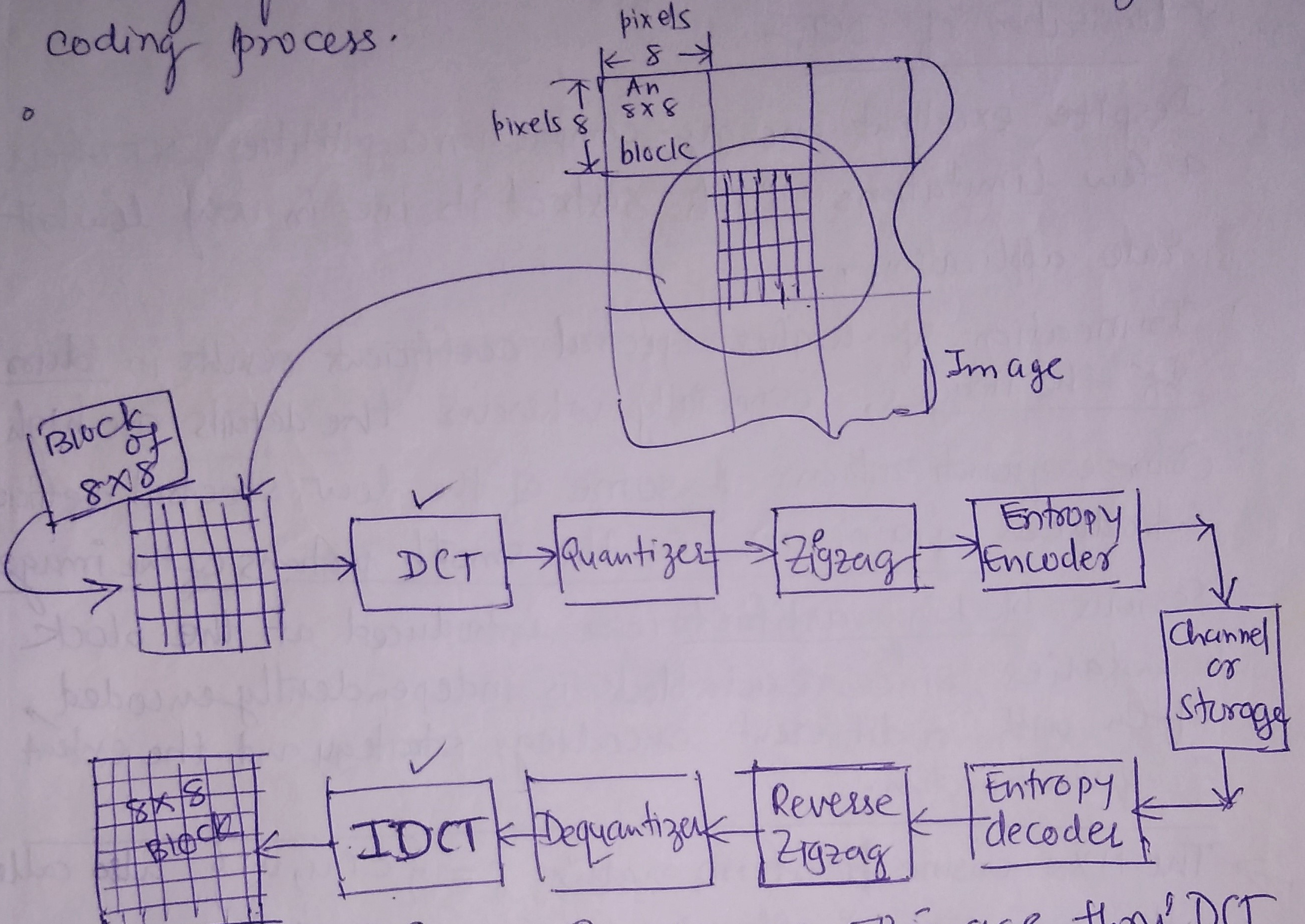

Fig:- Compression process of image thru' DCT.

DCT-based codecs use a two-dimensional version of the transform.

The 2D DCT and its inverse (IDCT) of an N×N block are shown below:-

2D DCT:-

$$F(u,v) = \frac{2}{N} C(u)C(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{(2x+1)u\pi}{2N}\right] . \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

2D IDCT:-

$$f(x,y) = \frac{2}{N} C(u) C(v) \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u,v) \cos\left[\frac{(2x+1)u\pi}{2N}\right] . \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

DCT is also decomposes a signal into a series of harmonic-cosine functions. (As in DFT).

★ Implementation issues of DCT:-

- Pre calculate the DCT coefficients and scale them.

★ Limitation of DCT:-

- Despite excellent energy compaction capabilities, DCT offers a few limitations which restrict its use in very low bit rate applications.
- Truncation of higher spectral coefficients results in blurring of the images, especially wherever the details are high.
- Coarse quantization of some of the low spectral coefficients introduces graininess in the smooth portions of the images.
- Serious blocking artifacts are introduced at the block boundaries, since each block is independently encoded, often with a different encoding strategy and the extent of quantization.

→ The N X N cosine transform matrix $C = \{c(u,v)\}$ also called the Discrete cosine (function) transform - DCT is defined as -

$$c(u,v) = \begin{cases} \frac{1}{\sqrt{N}} & ; u=0,\ 0 \le v \le (N-1), \\ \sqrt{\frac{2}{N}} \cdot \cos\left[\frac{\pi(2v+1)u}{2N}\right] & 1 \le u \le (N-1),\ 0 \le v \le (N-1) \end{cases}$$

$; 0 \le v \le 3$

$1 \le u \le 3$

for $N \times N = 4 \times 4$

$c(0,0) = \frac{1}{\sqrt{4}} = 0.5$

$c(0,1) = \frac{1}{\sqrt{4}} = 0.5$

$c(0,2) = \frac{1}{\sqrt{4}} = 0.5$

$c(0,3) = \frac{1}{\sqrt{4}} = 0.5$

$$\begin{array}{c|cccc} & v=0 & 1 & 2 & 3 \\ \hline u=0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 1 & 0.65 & 0.27 & -0.27 & -0.65 \\ 2 & 0.5 & -0.5 & -0.5 & 0.5 \\ 3 & 0.27 & -0.65 & 0.65 & -0.27 \end{array}$$

$c(1,0)$
$c(1,1)$
$c(1,2)$
$c(1,3)$ } to be contd. -

on Next page (25)

Pl. refer

$$C(u,v) = \sqrt{\frac{2}{N}} \cos \frac{\pi(2v+1)u}{2N} \; ; \text{ for } 1 \le u \le N-1$$

$$C(1,0) = \sqrt{\frac{2}{4}} \cos\left(\frac{\pi(0+1)\cdot 1}{2\times 4}\right)$$ for all u; $1 \le u \le 3$, v; $0 \le v \le 3$

$$C(1,0) = \sqrt{\frac{1}{2}} \cdot \cos\left(\frac{\pi}{8}\right) = +0.6532$$

$$C(1,1) = \sqrt{\frac{2}{4}} \cos \frac{\pi(2+1)1}{2(N=4)} = \sqrt{\frac{1}{2}} \cos\left(\frac{3\pi}{8}\right) = 0.2705$$

$$C(1,2) = \sqrt{\frac{2}{4}} \cos \frac{\pi(4+1)\cdot 1}{8} = \sqrt{\frac{1}{2}} \cdot \cos\left(\frac{5\pi}{8}\right) = -0.2705$$

$$C(1,3) = \sqrt{\frac{2}{4}} \cos \frac{\pi(6+1)\cdot 1}{8} = \sqrt{\frac{1}{2}} \cos\left(\frac{7\pi}{8}\right) = -0.6532$$

Illy
$C(2,0) = 0.5$
$C(2,1) = -0.5$
$C(2,2) = -0.5$
$C(2,3) = 0.5$

Illy
$C(3,0) = 0.2705$
$C(3,1) = -0.6532$
$C(3,2) = 0.6532$
$C(3,3) = -0.2705$

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* Properties of DCT

1. The cosine transform is real and orthogonal; ie
$$C = C^* = C^{-1} = C^T$$
2. The cosine transform is not the real part of the unitary DFT. However cosine transm of a sequence is related to the DFT of its symmetric extension.
3. The cosine transform is fast transform. Cosine transm of a vector 'N' elements can be calculated $(N \log_2 N)$ operations via 'N' pt. FFT.
4. The cosine transform has excellent energy compaction for highly correlated data.
5. Many of the coefficients are small, ie most of the energy of data is packed in a few transform coefficients.