★ Haar Transform :-

Algorithm.

The Haar functions $h_K(x)$ are defined on a continuos interval, $x \in [0, 1]$ and for $K = 0$ to $N-1$ (N-1), where $N = 2^n$.

The integer 'K' can be uniquely decomposed as

$$\boxed{K = 2^p + q - 1}$$

where $0 \le p \le n-1$; $q = 0, 1$ for $p = 0$

and $1 \le q \le 2^p$; for $p \neq 0$ for eg. when, $p > 0$

N=4 we have,

K	0	1	2	3
p	0	0	1	1
q	0	1	1	2

∴ Representing 'K' by (p, q) the Haar functions are defined as, $h_k(x) = h_{p,q}(x)$ for $x \in [0\ 1]$

for eg:-

$$h_0(x) \triangleq h_{00}(x) = \frac{1}{\sqrt{N}} \quad ; x \in [0, 1]$$

Pl. refer (

STFT: Short time fourier transform :-

★ Sinusoid with a fixed length instead of $\boxed{-\infty \text{ to } +\infty}$, it is refered as STFT.

★ F.T. offer sinusoid with window is called STFT.

★ Time - Frequency localization depends on window size.

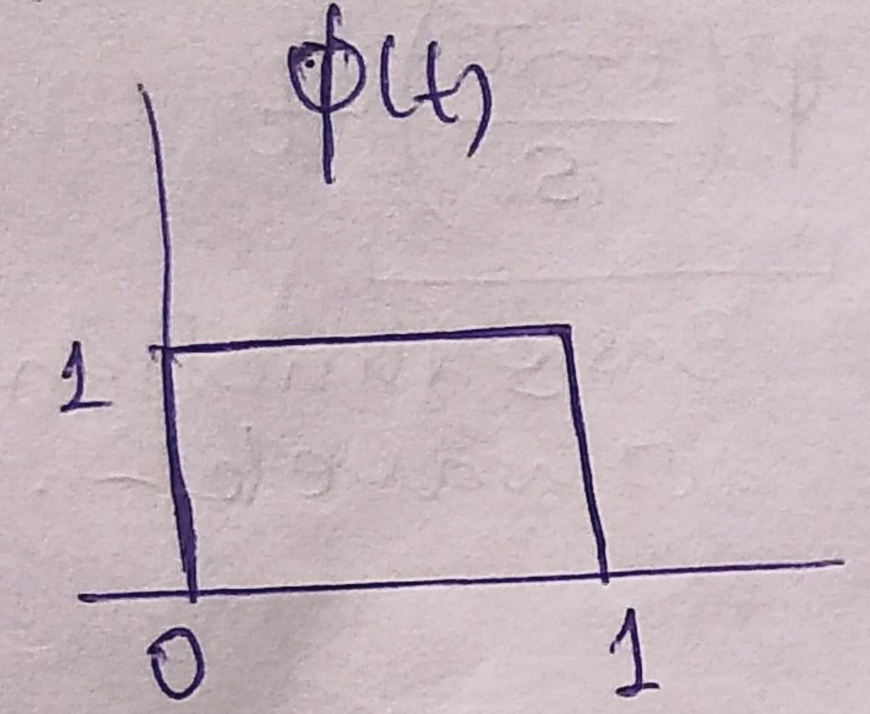
★ Time information is not visible/understand in F.T. for one particular frequency.

★ Wave-let family gives time - frequency information in particular window.

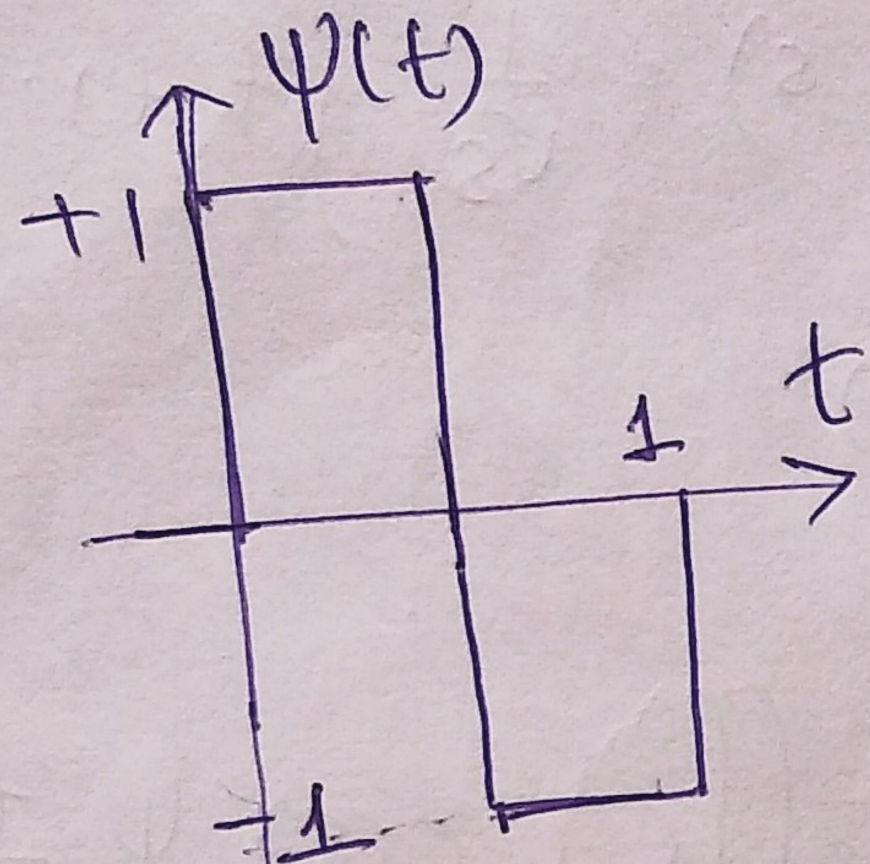
- The Haar transform is based on a class of orthogonal matrices, whose element are either 1, −1, or 0 and multiplied by powers of $\sqrt{2}$.

- The Haar transform is computationally efficient transform as the transform of N-point vector requires only $2(N-1)$ additions and multiplications.

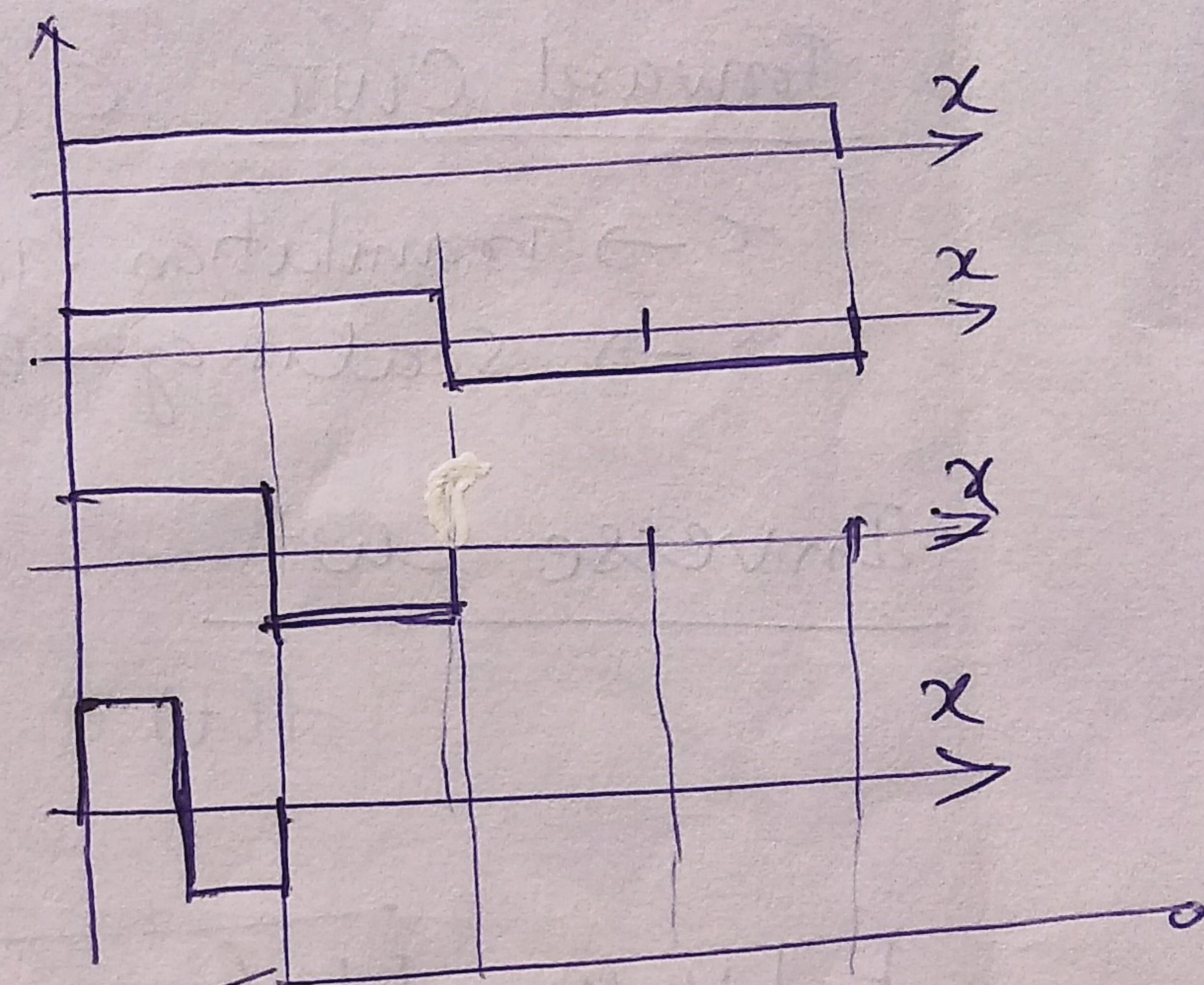
Haar scaling and Wavelet functions:-



Computes averages (low pass)



Computes details (high pass)



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HAAR Algo:

$$h_K(x) = h_{p,q}(x) = \begin{cases} \frac{1}{\sqrt{N}} 2^{p/2} ; & \text{for } \frac{q-1}{2^p} \le x < \frac{q-\frac{1}{2}}{2^p} \\ -\frac{1}{\sqrt{N}} 2^{p/2} ; & \text{for } \frac{q-\frac{1}{2}}{2^p} \le x < \frac{q}{2^p} \\ 0 & ; \text{for other } x \in [0,1] \end{cases}$$

$n = \log_2 N$

$n = 3 = \log_2 N$

for $N = 8$

$N = 2^3 = 8$

$$H_K(x) = \frac{1}{\sqrt{8}} \begin{array}{c} \\ \left[\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{array}\right]_{8\times 8} \end{array} \begin{array}{c} \text{Sequency} \\ 0 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array}$$

$\Updownarrow$

$H_{p,q}(x)$ (Haar basis function.)

The transform matrix 'H' is obtained by letting 'x' take discrete values at m/N, $(m = 0, 1, \ldots N-1)$

For $N = 8$:

$x \to \{0, 1\} \Rightarrow \{0/8, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, 6/8, 7/8\}$

- Real and Orthogonal
- transition at each scale 'p' is localized according to 'q'.
- Basis images of 2D (separable) Haar Transform
- Outer product of two basis vectors.

Properties (Haar Transm)

- Scaling captures information at different frequencies.
- Translation captures information at different locations.
- Can be represented by filtering and downsampling.
- Relatively poor energy compaction.
- Haar Tran. is real and orthogonal ∴ $H_r = H_r^*$
- Haar Trans. is very fast transform. On an $N \times 1$ vector it can be implemented in $O(N)$ operations.
- The basis vectors of the Haar matrix are sequency ordered.

$n = 3, N = 8$ $\quad k = 2^p + q - 1$

$n = 2;\ N = 4$

$k =$	0	1	2	3	4	5	6	7
$p =$	0	0	1	1	2	2	2	2
$q =$	0	1	1	2	1	2	3	4

$0 \le p \le n-1$, $q = 0, 1$ for $p = 0$

$1 \le q \le 2^p$ $\quad p \ne 0, p > 0$

$-1 \le x < -\frac{1}{2}$	$p = q = 0$
$-\frac{1}{2} \le x \le 0$	$p = q = 0$

when $k = 1$, $p = 0$, $q = 1$

Conditions:-

(i) $0 \le x \le \frac{1}{2}$

(ii) $\frac{1}{2} \le x \le 1$

(iii) Otherwise $H_1(x) = 0$

when $k = 0$, $p = 0$, $q = 0$ Conditions

(i) $-1 \le x < -\frac{1}{2}$

(ii) $-1/2 \le x < 0$

(iii) Otherwise $H_0(x) = 0$

* Sample Calculations : Haar Transforms

for $N=8$; $m = 0$ to $N-1 = 0$ to 7

$x \to m/N \to 0/8, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8$.

Case ① If $k=0$, $p=q=0$, the $h_0(x) = \frac{1}{\sqrt{8}}$ for all $x \to 0$ to $\frac{7}{8}$

Case ② when

$k=1$, $p=0$, $q=1$

(a) for $x=0$, the first condition is satisfied.

i.e. $0 \le x < \frac{1}{2}$ $\quad H_{k=1}(x) = H_1(x) = \frac{1}{\sqrt{N}} 2^{p/2} = \frac{1}{\sqrt{8}} 2^{0/2} = \frac{1}{\sqrt{8}}$

(b) for $x = 1/8$, I condition
(c) for $x = 1/4$, I condition
(d) for $x = 3/8$, I condition

satisfied. Hence all $H_1(x) = \frac{1}{\sqrt{8}}$ upto $x = 3/8$

Now (e) $x = \frac{1}{2}$ the second condition is satisfied.

i.e. $\frac{1}{2} \le x < 1$ $\quad H_1(x)_{x=\frac{1}{2}} = H_1(\frac{1}{2}) = \frac{-1}{\sqrt{N}} 2^{0/2} = \frac{-1}{\sqrt{8}}$

(f) for $x = 5/8$, $3/4$ and last value $x = 7/8$ the

to (h) condition II is confirm and satisfied.

so for $H_1(x) = \frac{-1}{\sqrt{8}}$.
$x = \frac{5}{8}, \frac{3}{4}, 7/8$

Case ③ when $k=2$, $p=1$, $q=1$

(a) for $x=0$, the I condition is satisfied.

$\frac{q-1}{2^p} \le x < \frac{q-\frac{1}{2}}{2^p} \Rightarrow 0 \le x < \frac{1}{4}$

$H_2(x)_{x=0} = \frac{1}{\sqrt{N}} 2^{p/2} = \frac{1}{\sqrt{8}} 2^{1/2} = \left(\frac{1}{\sqrt{8}}\right) \sqrt{2}$

IIIly evaluate all the elements of 8×8 matrix